The Decoy Effect and Risk Aversion

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Abstract

The decoy effect arises when a firm offers a product that is clearly inferior to another product in order to drive sales of the latter. This phenomenon has been displayed experimentally in many different situations, but remains understudied theoretically. We develop a model of almost rational consumer choice, with a single behavioral tendency — regret aversion. Consumers aim to maximize their expected utility in a situation where they face incomplete information. We derive probability consumer choice in a setting with signal noise, where they are uncertain about the qualities of their alternatives. They display regret aversion, in that if the consumer later on discovers they have not selected the option that maximizes their ex-post utility, they face an additional utility loss. In this paper, we demonstrate that regret aversion is sufficient to generate the decoy effect in most cases, but does not do so for all possible "decoys". We then use our findings to derive several testable implications that enable experimenters to determine for themselves whether or not regret aversion is the ultimate generator of the decoy effect in practice. Finally, we describe practical implications of our theory for firms that have reason to believe regret aversion is driving their consumer towards decoyed alternatives.

1 Introduction

To illustrate how the decoy effect works, consider a study by Geographic (2018), which poses a scenario featuring two options of popcorn sizes with different prices in a movie theatre.

- Small popcorn for $3.
- Large popcorn for $7.

Depending on their appetite, customers' decisions varied and were difficult to predict. Yet, most people in the experiment chose the small size because the large size seemed too big to finish during a single movie. The same experiment was conducted after adding a third option, a medium popcorn size option for $6.50.

- Small popcorn for $3
- Medium for $6.50
- Large popcorn for $7

People now chose the large size because paying only a half a dollar more to purchase the large size seemed to be a better deal. Adding the medium size, as a result, helped people to perceive value in the large popcorn option that they did not see previously. The decoy effect was exhibited through the introduction of the third option.

As this example suggests, the decoy effect is a "phenomenon whereby the consumer's preferences between two options tend to change when presented with a third option that is asymmetrically dominated" (Mortimer, 2019). As Mortimer (2019) asserts, asymmetric domination refers to a situation in which the decoy is priced to make one of the options more attractive in perceived value (e.g., quantity or quality). In other words, the decoy itself is not meant to be sold: instead, its primary function is to nudge consumers away from the competitor and towards the target — usually the more expensive or profitable option for the firm.

Many industries leverage this nudging strategy —"a choice structure that predictably changes people's behaviour" — to encourage customers to purchase the products that are more profitable for the firms (Vivekanandarajah, 2020). For instance, newspapers and magazines often price their subscription plans in a way that attempts to capitalize on the decoy effect (Hendricks, 2018).

Consider *The Economist*, a magazine that focuses on modern economic and political challenges. The company offers three subscription options: $59 for electronic only, $125 for print only, and $125 for print and electronic. When weighing only the first and third options, most customers choose the first option due to its lower price. However, when the second option is added, it makes the third option asymmetrically dominant relative to the other two, swaying consumers to choose the third option. As this example illustrates, the decoy effect increases the predictability of consumer behavior, giving firms who incorporate it into their pricing strategies a clear upper hand.

Psychologist Barry Schwartz asserts that when consumers are faced with many alternatives, they often experience choice overload, which Schwartz refers to as "the tyranny or paradox of choice" (Mortimer, 2019). Multiple behavioral experiments have consistently demonstrated that greater choice hinders decision-making, as consumers face greater anxiety over making the correct choice. In an attempt to reduce this anxiety, consumers tend to compare the given options on only a few criteria — most often price, quantity, and quality — to make the best decision that maximizes their utility. A decoy, therefore, manipulates consumers into acting in a way that leads them to believe they are making a rational, informed choice based on the perceived qualities of the given options (Mortimer, 2019). In other words, the decoy effect may utilize regret to steer consumers towards a particular choice.

This paper will focus upon the ability of regret aversion to generate the decoy effect. Regret aversion can be defined as a pattern of consumer behavior in which individuals endeavor to avoid putting themselves in a situation where hindsight indicates they had a better option. When making decisions, one should consider all outcomes, as well as the likelihood and effect of each outcome, before reaching a final verdict. In practice, decision makers evaluate their choices in a far different manner. There is a growing body of evidence that suggests a substantial proportion of such decision makers think about the worst possible outcome and how they would feel about that outcome in terms of their level of regret. They then choose the option that minimizes regret, even if it is not optimal from an ex-ante perspective.

Prospect theory, as developed extensively in Tversky and Kahneman (1992) provides one possible justification for regret aversion. Prospect theory dictates that individuals have standard utility functions except at an arbitrarily defined "reference point." At this reference point, the utility function features a kink, where losses are felt more strongly than gains. In our context, regret aversion is then a specific form of prospect theory, where the reference point is the consumer's optimal utility. Falling short of this optimum causes the consumer to overvalue their shortcoming.

In general, the literature comments on the presence of the decoy effect in practice, but fails to generate a behavioral assumption that implies it. One key exception is the salience theory framework, in which the decoy can "draw attention" to one particular quality, thereby boosting the attractiveness of goods with a high level of that quality. However, there has not been a satisfactory theory that attempts to address the following question: Can regret aversion explain the decoy effect in practice? If so, what implications could be drawn that would distinguish it from the existing literature?

This paper proceeds as follows. Section 2 explores past research related to the decoy effect. In Section 3, we outline the model that we will use to demonstrate the correlation between regret aversion and the decoy effect. Section 4 derives our results and proofs, and Section 5 summarizes our findings and offers potential avenues for future research.

2 Literature Review

Our research touches on several fields of literature, primarily behavioral and experimental.

First, any such section on the decoy effect would be incomplete without mentioning Huber et al. (1982), one of the first papers to formally describe what we informally refer to as the decoy effect. In Huber et al. (1982), the authors showed that the current choice models of the period failed to adequately address the presence of the decoy effect.

We find that regret aversion can play a key role in generating the decoy effect. One early paper that describes this effect in order to better understand consumer behavior is Bell (1982). We will focus on a simplified version of the model in Loomes and Sugden (1982). Their paper experimentally tests various predictions of regret aversion, comparing it to the standard model of consumer preferences. They show that the standard model fails to predict certain consistent errors in consumer choice. Their model enables us to better describe consumer choice behavior. In both cases, we are considering a single type of behavior as opposed to the more broad characterizations those papers consider. These papers are largely spawned by the literature that followed the original Tversky and Kahneman (1981) paper on prospect theory.

On a similar note, salience theory as described in Bordalo et al. (2013) could also have been used to generate the decoy effect. It is worth noting that both models will predict that the decoy effect is not necessary for the "nudging strategy" to work effectively. If instead there were two objects similar in the two dimensions of interest but explicitly different from a third object, the two objects would increase in choice frequency under both theories of choice. This is a potential flaw in the theory, but one that should be made explicit. One major difference in the implication of our model and theirs is that our effect is diminishing in the distance between the decoy and the target. Specifically, if the decoy is too low in both quality dimensions, and is clearly inferior to both objects, it will not have an effect — salience theory doesn't necessarily make the same predictions.

Last, Sachley (2005) explores a simple model of consumer flux and shows that regret is not necessary for the decoy effect to emerge. The paper uses a reduced form analysis taking the flux as given without considering the underlying assumptions that generate such flux. The primary difference in their methodology and ours is that Sachley (2005) doesn't provide an ultimate justification for why flux appropriately describes consumer behavior. That is, consumers are assumed to feature a specific irrationality in their behavior, but that irrationality is arbitrarily chosen. In this paper, we focus on a consumer that faces an informational constraint restricting their ability to learn too much about each object.

3 Model

We model a single decision maker who will choose an element x in a set of alternatives X. The decision maker aims to choose the object that maximizes her utility function. Her utility is additive in two terms: the direct utility from the chosen object and the regret ex-post from observing the value of better objects not chosen.

Each element x of X has two qualities — we will use x_1 and x_2 to denote these. For example, x_1 might represent the price level of good x (we will assume a higher x_1 is better) while x_2 represents the quality of good x. $v(x_1, x_2)$ then signifies the decision maker's valuation of an object with qualities x_1, x_2.

At the same time, the decision maker faces regret if she makes mistakes. After choosing object x, she then learns her exact valuation of the remaining objects and pays a cost $-c$ for each object she would have preferred to her choice. That is if she chooses object x and learns that there were two better objects in the end her utility is $u = v(x_1, x_2) - 2c$.

Crucially, the decision maker is not initially aware of each good's qualities: before deciding which alternative to consume, she receives noisy signals of each of the good's qualities (x_1^s, x_2^s). Both of these signals are uniformly distributed around x_i. That is, $x_i^s \sim U[x_i - \delta, x_i + \delta]$, where U is the uniform distribution and $\delta > 0$ is a noise parameter. These two signals, and signals of other goods' qualities are all drawn independently.

In this paper, we will focus on the case where there are two alternatives and consider the change in selection probabilities when a third good is added. These alternatives will be denoted by x, y, and z respectively. In particular, we are interested in the case where z is "dominated" by x in a sense we will later formally define.

A useful image to display these choice probabilities is a probability simplex. If there are n alternatives, we can depict the probability the decision maker chooses an alternative on a $n-1$ dimensional simplex. For example, if there exist two products, then the line segment connecting $(0,1)$ to $(1,0)$ can be used to represent decision probabilities. $(2/3, 1/3)$ signals that the decision maker will choose the first alternative with probability $2/3$ and the second with probability $1/3$. This concept (if not the image) easily extends to higher dimensions as well. Formally, the decision maker's decision probability is given by $(k_1, k_2, \ldots, k_{|X|})$ where $\sum k_i = 1, k \geq 0$. This directly describes choice probabilities conditional on utilities.

For instance, Figure S1 displays a probability simplex when $|X| = 3$. Each vertex of the triangle indicates that a certain good is always chosen with probability 1. Intermediate points illustrate that from the perspective of an outsider observer the decision maker is randomizing over the goods with positive probability.

This element of randomization is an attractive feature of our model, as a model that always predicts a certain good is chosen with probability one can't be rationalized by any reasonable data set. Instead, our model's parameters can be adjusted based on the data set used as well as certain reasonable restrictions, improving its testability.

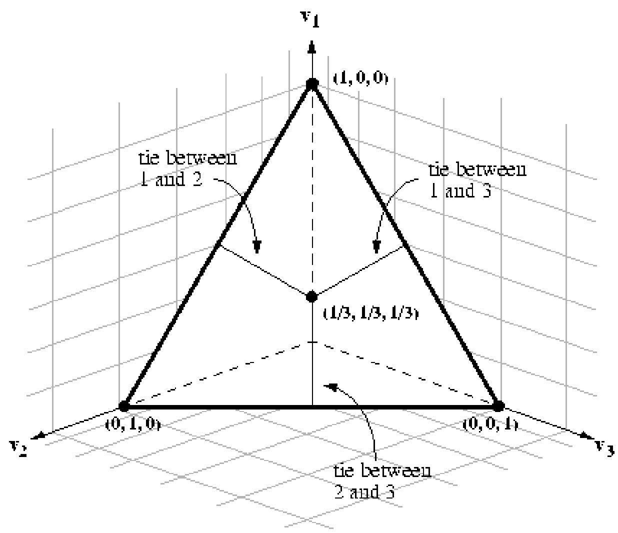
4 Results

To simplify the math in this section, we make the following assumption.

Assumption 1. $v(x_1, x_2) = x_1 x_2$

We conjecture that Assumption 1 is not necessary for our results, and that any smooth, quasi-concave function of x_1 and x_2 will also generate our results. Such an assumption would be in-line with standard

FIGURE S1: **Three Good Choice Simplex Example**



models of consumer behavior. Of course, the above function becomes unreasonable when $x_1, x_2 < 0$, implying sudden discontinuities in utility. In particular, it doesn't appear reasonable for a quality to "appear negative," so to avoid running into edge cases, we focus on the following parameter region:

Assumption 2. $x_i > \delta$

Assumption 2 implies that the consumer always knows that both qualities are positive and never expects otherwise. We proceed by considering the simplest case, with two alternatives, x and y, that have qualities (x_1, x_2) and (y_1, y_2) respectively. We determine the choice probabilities in this setting. In order to do so, we begin by breaking the problem down into simpler steps.

- First, taking the signal realizations as given, we infer the decision maker's optimal choice.
- Second, we show that the signal generating structure doesn't qualitatively impact the decision maker's valuation.
- Finally, taking a step back, we compute the probabilities of each signal realization. This enables us to determine the ex-ante choice probabilities of the consumer.

4.1 2 Product: Ex-Post Decision Making

In an extreme situation where one alternative has both signal realizations of quality appear substantially above the signal realizations of the other alternative's quality, the former alternative will always be chosen. To see this, consider the decision maker's problem. She aims to maximize her utility while avoiding regret, which is equivalent to $\max_{k \in X} k_1 k_2$ when there are only two alternatives. It immediately follows that the object with greater posteriors in both is the better alternative.

Lemma 1. *When there are only two alternatives, if one good has both signal realizations of quality above both signal realizations of the other good, the former will be chosen by the decision maker.*

If both alternatives have one signal realization that is greater than one of the other's signal realizations, then a bit of algebra is required. She computes the gain from each good as $\mathbf{E}[k_1 k_2]$, which given her signals, requires her to invert two uniform variables. In this case, it is given by $\int_{k_1^s-\delta}^{k_1^s+\delta} \int_{k_2^s-\delta}^{k_2^s+\delta} (\frac{1}{2\delta})^2 s_1 s_2 ds_2 ds_1$, which is equivalent (up to a constant) to $\int_{k_1^s-\delta}^{k_1^s+\delta} \int_{k_2^s-\delta}^{k_2^s+\delta} s_1 s_2 ds_2 ds_1$. She then picks the object that has the greater such value. We simplify and find that $\int_{k_1^s-\delta}^{k_1^s+\delta} \int_{k_2^s-\delta}^{k_2^s+\delta} s_1 s_2 f(s_1, s_2) ds_2 ds_1 = k_1^s k_2^s$[1].

Lemma 2. *The expected value of an alternative with signal realizations x_1^s, x_2^s is $x_1^s x_2^s$. It follows that when there are two alternatives, the decision maker chooses the object such that the product of its signals is greater than the product of the other alternative's signals.*

Then, Lemma 2 implies two corollaries. First, our choice of noise distribution doesn't fundamentally impact the resulting posterior. In particular, the fact that the expected value mimics the initial value function corroborates this statement. Second, this considerably simplifies the following analysis, as it provides a simple functional form with which the decision maker's choice can be described.

Next, when two objects are "close" to each other in terms of their valuations, we need to determine the probability that each has a higher expected value based on the noise distribution. In order to do so, the following calculation will be useful. We compute the Cumulative Distribution Function of a product of uniform distributions:

$$\frac{1}{2\delta}(\frac{k}{x_1+\delta} - (x_2-\delta)) + \frac{1}{4\delta^2}[kln(x_2+\delta) - kln(\frac{k}{x_1+\delta}) - (x_2+\delta)(x_1-\delta) + \frac{k}{x_1+\delta}(x_1-\delta)]$$

Next, we take the derivative of this term to calculate the PDF:

$$f(k) = \frac{1}{2\delta}\frac{1}{x_1+\delta} + \frac{1}{4\delta^2}\left[ln(x_2+\delta) - ln(\frac{k}{x_1+\delta}) - (x_1+\delta) + \frac{x_1-\delta}{x_1+\delta}\right]$$

We summarize this result in the following technical lemma:

Lemma 3. *The PDF of a product of uniform distributions is given by:*

[1] All calculations and proofs are relegated to the appendix

$$\frac{\partial \mathbf{P}(x_1^s x_2^s \leq k)}{\partial k} = \frac{1}{2\delta}\frac{1}{x_1+\delta} + \frac{1}{4\delta^2}\left[ln(x_2+\delta) - ln(\frac{k}{x_1+\delta}) - (x_1+\delta) + \frac{x_1-\delta}{x_1+\delta}\right]$$

Since the signals of the two goods are independent, we can take the joint density function of $f(x_1x_2, y_1y_2)$ as the product of two copies of the previous PDF: $f(x_1x_2, y_1y_2) = f(k_1)f(k_2)$. This will be necessary for the following computation.

The ultimate goal is to determine the exact probability with which an alternative appears to be preferred to another alternative. We first rule out the case where one alternative clearly dominates the other in order to simplify our consideration. When the best case scenario for one good still yields lower expected utility than the worst case scenario for the other good in terms of noise, the former will never be chosen.

Definition 1. *Alternative x is clearly inferior to alternative y if* $(x_1+\delta)(x_2+\delta) < (y_1-\delta)(y_2-\delta)$

Remark 1. *If there are only two alternatives, and if x is clearly inferior to y, then alternative x is chosen with probability* 0*.*

More generally, any set of alternatives that includes clearly inferior options can be reduced to a set excluding those options. However, doing so may change the eventual choice probabilities due to the modelling choice of regret aversion. Consider a scenario in which alternative x is clearly inferior to alternative y, but not clearly inferior to alternative z. If the decision maker was previously selecting y, z with positive probability, she must now select alternative z with smaller probability, as there exists positive probability from the decision maker's perspective that z was not only inferior to y, but also to x, therefore causing an increased level of regret and making her less willing to choose z in general.

Lemma 4. *Let X denote a set of alternatives, with $|X| > 2$. There exist sets of qualities for the alternatives in X such that the choice probabilities over X are not equivalent to those over a subset with clearly inferior alternatives in X removed.*

In the remainder of this paper, we focus on alternatives that are closer to each other in respective utility.

Assumption 3. $(x_1+\delta)(x_2+\delta) \geq (y_1-\delta)(y_2-\delta)$ *and* $(y_1+\delta)(y_2+\delta) \geq (x_1-\delta)(x_2-\delta)$*. That is, x is not clearly inferior to y, and y is not clearly inferior to x.*

Lastly, we compute the probability that the expected utility of x is greater than the expected utility of y using a multivariate integration over the respective probabilities of noise. Here we take advantage of lemma 3 to simplify the computation.

$$
\int_{x_1-\delta,x_2-\delta}^{x_1+\delta,x_2+\delta}\int_{y_1-\delta,y_2-\delta}^{y_1+\delta,y_2+\delta}(\frac{1}{2\delta}\cdot\frac{1}{x_1+\delta}+\frac{1}{4\delta^2}\cdot\left[ln(x_2+\delta)-ln(\frac{k}{x_1+\delta})-(x_1+\delta)+(\frac{x_1-\delta}{x_1+\delta})\right])
$$
$$
\times(\frac{1}{2\delta}\cdot\frac{1}{y_1+\delta}+\frac{1}{4\delta^2}\cdot\left[ln(y_2+\delta)-ln(\frac{k}{y_1+\delta})-(y_1+\delta)+(\frac{y_1-\delta}{y_1+\delta})\right])\cdot d(y_1y_2)d(x_1x_2)
$$

Theorem 1. *When there are two goods, with qualities $(x_1, x_2), (y_1, y_2)$ and assumption 3 is satisfied, good x is chosen with probability:*

$$
\left(\frac{(ln\frac{y_1+2\delta}{y_1})}{2\delta}+\frac{\frac{(y_2+2\delta)ln(y_2+2\delta)-(y_2+\delta)}{(y_2)ln(y_2)-(y_2-\delta)}}{4\delta^2}+\frac{\frac{2(y_1)ln(|y_1+2\delta|)-(y_1+\delta)^2-2(ln(k)+\delta)(y_1+\delta)}{2(y_1)ln(|y_1|)-(y_1-\delta)^2-2(ln(k)+\delta)(y_1-\delta)}}{8\delta^2}\right)
$$
$$
\cdot\left(\frac{(ln\frac{x_1+2\delta}{x_1})}{2\delta}+\frac{\frac{(x_2+2\delta)ln(x_2+2\delta)-(x_2+\delta)}{(x_2)ln(x_2)-(x_2-\delta)}}{4\delta^2}+\frac{\frac{2(x_1)ln(|x_1+2\delta|)-(x_1+\delta)^2-2(ln(k)+\delta)(x_1+\delta)}{2(x_1)ln(|x_1|)-(x_1-\delta)^2-2(ln(k)+\delta)(x_1-\delta)}}{8\delta^2}\right)
$$

4.2 Three Product Subcase

Now, we can finally move on to the decoy effect itself. In this section, we will refer to the decoy as alternative z.

To simplify the analysis, we consider two extreme cases. First, let z be clearly inferior to x and y for any signal realizations. Then, we argue that the presence of z leaves the problem unchanged for the decision maker. It clearly cannot affect the problem through regret aversion and it can never appear superior to either good.

Proposition 1. *If there are three goods, x, y, z, and for any signal realization, z is clearly inferior to both x and y, then the choice probabilities in X are the same as in $X \setminus \{z\}$.*

To map this result back onto the decoy effect, consider a firm was attempting to use product z as a decoy for product x in order to dissuade consumers from choosing product y, meaning that $x-\delta > z+\delta$ (component by component). This result would offer the specific instruction that z must be comparable to y; otherwise, its presence cannot induce the consumer to choose product x with higher probability. If y is too promising of a product relative to z, then the introduction of the decoy will have no effect. Harkening back to the previous example of magazine subscriptions, if the consumer would never choose to purchase just the print version for the stated price, then its inclusion as an option will not increase the probability with which the consumer chooses to purchase the print/online bundle.

While the specifics of this result depend upon the distribution of noise utilized, we argue that it holds in general as long as the noise distribution does not have a "fat tail". In other words, if the consumer doesn't place a large probability weight that they are exceptionally wrong in their estimation of a product, then the natural extension of this result will hold. Rather than the clearly inferior product having 0 impact, it will instead have a small, but non-trivial impact upon the consumer's choice probabilities.

Contrast this with the case where the alternatives x, z are exactly identical to each other. Then, the decision maker will be far less willing to choose option y. In effect, the probability of regret doubles itself. There are two separate effects at play here: first, due to sheer probability, it is more likely that option y appears dominate. Second, the presence of option z decreases the expected utility from option y, as the decision maker believes there is a chance that option y is inferior to two separate options, leading to a direct increase in regret.

Theorem 2. *Let there be three goods, x, y, z, where x and z have identical qualities, and no good is clearly inferior to any other good. Then y's choice probability in the set of alternatives $\{x, y, z\}$ is strictly less than in $\{x, y\}$.*

This theorem and proposition 1 combine to imply that a utilized decoy cannot be clearly inferior to both alternatives in order to sway the consumer.

5 Conclusion

In this paper, we have analyzed the relationship between regret aversion and the decoy effect, eventually arriving at the conclusion that regret aversion does generate the decoy effect, but does so in a very specific manner. A purported decoy cannot be clearly inferior to the options it is attempting to influence; otherwise, it will have no effect. By contrast, a decoy that is only slightly worse than the option it is buoying will have the strongest effect. However, this implies that sometimes such decoys will be selected instead. Indeed, such decoys could potentially be considered variants of the original product rather than decoys. This line of reasoning provides a justification for why companies invest so heavily in variations of their products. For example, consider Coca Cola, which includes over 3,500 different types of beverages in its lineup. By providing many different products that appear to be close in value, Coca Cola subtly pushes consumers towards purchasing one of its products rather than those of its competitors.

Notably, while we assume specific functional forms for the quantities present, the associated assumptions are not particularly strong. We assumed that signals were bounded through the introduction of δ and that consumer utility was quasi-concave and multiplicative in our functional form. Both of these assumptions are easily justified. We require a bound on the signal structure to prevent signals from appearing negative. Similarly, consumer utility has been shown to satisfy quasi-concavity through numerous experimental studies.

The theory examined in this paper provides several clearly testable predictions. If there exists some form of continuity and monotonicity in quality, the shift in choice probabilities induced by a decoy will be decreasing in its difference with the pushed alternative. That being said, this could potentially conflict with a form of choice paralysis wherein two alternatives that appear to be too close to each other cause the decision maker to choose something else instead. We leave this testing and theoretical implication for future research.

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A Appendix – Proofs

Expected utility conditional on signal calculation

$$\begin{aligned}
\int_{k_1^s-\delta}^{k_1^s+\delta}\int_{k_2^s-\delta}^{k_2^s+\delta} s_1 s_2 f(s_1,s_2) ds_2 ds_1 &= \frac{1}{4\delta^2}\int_{k_1^s-\delta}^{k_1^s+\delta} (s_1 s_2^2/2|_{k_2^s-\delta}^{k_2^s+\delta} ds_1 \\
&= \frac{1}{4\delta^2}\int_{k_1^s-\delta}^{k_1^s+\delta} s_1/2[(k_2^s+\delta)^2-(k_2^s-\delta)^2] \\
&= \frac{1}{4\delta^2}\int_{k_1^s-\delta}^{k_1^s+\delta} 2 s_1 k_2^s \delta \\
&= \frac{1}{4\delta^2} 2k_2^s\delta 2k_1^s\delta \\
&= \frac{1}{4\delta^2} 4\delta^2 k_1^s k_2^s \\
&= k_1^s k_2^s
\end{aligned}$$

Cumulative Distribution of a product of uniform distributions Note that $f(x_1^s) = 0$ if $x_1^s \notin [x_2-\delta, x_2+\delta]$.

$$\begin{aligned}
\mathbf{P}(x_1^s x_2^s \leq k) &= \int_{-\infty}^{\infty} \mathbf{P}(x_1^s \leq \frac{k}{z}) f(x_1^s) dz \\
&= \int_{x_2-\delta}^{x_2+\delta} \mathbf{P}(x_1^s \leq \frac{k}{z}) f(x_1^s) dz \\
&= \int_{x_2-\delta}^{\frac{k}{x_1+\delta}} \frac{1}{2\delta} dz + \int_{\frac{k}{x_1+\delta}}^{x_2+\delta} \frac{k/z - (x_1-\delta)}{2\delta} \frac{1}{2\delta} dz \\
&= \Big(\frac{z}{2\delta} \Big|_{x_2-\delta}^{\frac{k}{x_1+\delta}} + \Big(\frac{kln(z) - z(x_1-\delta)}{2\delta} \cdot \frac{1}{2\delta} \Big|_{\frac{k}{x_1+\delta}}^{x_2+\delta} \\
&= \frac{1}{2\delta}(\frac{k}{x_1+\delta} - (x_2-\delta)) + \frac{1}{4\delta^2}[kln(x_2+\delta) - kln(\frac{k}{x_1+\delta}) \\
&\quad - (x_2+\delta)(x_1-\delta) + \frac{k}{x_1+\delta}(x_1-\delta)]
\end{aligned}$$

Probability $\mathbf{E}[u(x)] > \mathbf{E}[u(y)]$ computation

$$\begin{aligned}
\mathbf{P}(\mathbf{E}[u(x)] > \mathbf{E}[u(y)]) &= \mathbf{P}(x_1^s x_2^s > y_1^s y_2^s) \\
&= \mathbf{P}(x_1^s x_2^s - y_1^s y_2^s > 0) \\
&= \int_{x_1-\delta, x_2-\delta}^{x_1+\delta, x_2+\delta} \int_{y_1-\delta, y_2-\delta}^{y_1+\delta, y_2+\delta} f(x_1^s x_2^s, y_1^s y_2^s) d(y_1 y_2) d(x_1 x_2) \\
&= \int_{x_1-\delta, x_2-\delta}^{x_1+\delta, x_2+\delta} \int_{y_1-\delta, y_2-\delta}^{y_1+\delta, y_2+\delta} (\frac{1}{2\delta} \cdot \frac{1}{x_1+\delta} + \frac{1}{4\delta^2} \cdot \Big[ln(x_2+\delta) - ln(\frac{k}{x_1+\delta}) - (x_1+\delta) + (\frac{x_1-\delta}{x_1+\delta}) \Big]) \\
&\cdot (\frac{1}{2\delta} \cdot \frac{1}{y_1+\delta} + \frac{1}{4\delta^2} \cdot \Big[ln(y_2+\delta) - ln(\frac{k}{y_1+\delta}) - (y_1+\delta) + (\frac{y_1-\delta}{y_1+\delta}) \Big]) \cdot d(y_1 y_2) d(x_1 x_2)
\end{aligned}$$

We factor out all terms that are constant with respect to y.

$$\int_{x_1-\delta, x_2-\delta}^{x_1+\delta, x_2+\delta} X \int_{y_1-\delta, y_2-\delta}^{y_1+\delta, y_2+\delta} (\frac{1}{2\delta} \cdot \frac{1}{y_1+\delta} + \frac{1}{4\delta^2} \cdot \Big[ln(y_2+\delta) - ln(\frac{k}{y_1+\delta}) - (y_1+\delta) + (\frac{y_1-\delta}{y_1+\delta}) \Big]) \cdot d(y_1 y_2) d(x_1 x_2)$$

We first solve the inner integration.

$$\begin{aligned}
&\int_{y_1-\delta, y_2-\delta}^{y_1+\delta, y_2+\delta} (\frac{1}{2\delta} \cdot \frac{1}{y_1+\delta} + \frac{1}{4\delta^2} \cdot \Big[ln(y_2+\delta) - ln(\frac{k}{y_1+\delta}) - (y_1+\delta) + (\frac{y_1-\delta}{y_1+\delta}) \Big]) \cdot d(y_1 y_2) \\
&= \Big(\frac{ln(|y_1+\delta|)}{2\delta} + \frac{(y_2+\delta)ln(y_2+\delta) - y_2}{4\delta^2} + \frac{2(y_1-\delta)ln(|y_1+\delta|) - y_1^2 - 2(ln(k)+\delta)y_1}{8\delta^2} \Big|_{y_1-\delta, y_2-\delta}^{y_1+\delta, y_2+\delta} \\
&= (\frac{ln(|y_1+2\delta|)}{2\delta} + \frac{(y_2+2\delta)ln(y_2+2\delta) - (y_2+\delta)}{4\delta^2} + \frac{2(y_1)ln(|y_1+2\delta|) - (y_1+\delta)^2 - 2(ln(k)+\delta)(y_1+\delta)}{8\delta^2}) \\
&- (\frac{ln(|y_1|)}{2\delta} + \frac{(y_2)ln(y_2) - (y_2-\delta)}{4\delta^2} + \frac{2(y_1)ln(|y_1|) - (y_1-\delta)^2 - 2(ln(k)+\delta)(y_1-\delta)}{8\delta^2}) \\
&= \frac{(ln\frac{y_1+2\delta}{y_1})}{2\delta} + \frac{\frac{(y_2+2\delta)ln(y_2+2\delta) - (y_2+\delta)}{(y_2)ln(y_2) - (y_2-\delta)}}{4\delta^2} + \frac{\frac{2(y_1)ln(|y_1+2\delta|) - (y_1+\delta)^2 - 2(ln(k)+\delta)(y_1+\delta)}{2(y_1)ln(|y_1|) - (y_1-\delta)^2 - 2(ln(k)+\delta)(y_1-\delta)}}{8\delta^2}
\end{aligned}$$

Now we solve the full integration.

$$\int_{x_1-\delta,x_2-\delta}^{x_1+\delta,x_2+\delta}(\frac{(ln\frac{y_1+2\delta}{y_1})}{2\delta}+\frac{\frac{(y_2+2\delta)ln(y_2+2\delta)-(y_2+\delta)}{(y_2)ln(y_2)-(y_2-\delta)}}{4\delta^2}+\frac{\frac{2(y_1)ln(|y_1+2\delta|)-(y_1+\delta)^2-2(ln(k)+\delta)(y_1+\delta)}{2(y_1)ln(|y_1|)-(y_1-\delta)^2-2(ln(k)+\delta)(y_1-\delta)}}{8\delta^2})$$
$$\cdot(\frac{1}{2\delta}\cdot\frac{1}{x_1+\delta}+\frac{1}{4\delta^2}\cdot\left[ln(x_2+\delta)-ln(\frac{k}{x_1+\delta})-(x_1+\delta)+(\frac{x_1-\delta}{x_1+\delta})\right])\cdot d(x_1x_2)$$
$$=(\frac{(ln\frac{y_1+2\delta}{y_1})}{2\delta}+\frac{\frac{(y_2+2\delta)ln(y_2+2\delta)-(y_2+\delta)}{(y_2)ln(y_2)-(y_2-\delta)}}{4\delta^2}+\frac{\frac{2(y_1)ln(|y_1+2\delta|)-(y_1+\delta)^2-2(ln(k)+\delta)(y_1+\delta)}{2(y_1)ln(|y_1|)-(y_1-\delta)^2-2(ln(k)+\delta)(y_1-\delta)}}{8\delta^2})$$
$$\cdot\int_{x_1-\delta,x_2-\delta}^{x_1+\delta,x_2+\delta}(\frac{1}{2\delta}\cdot\frac{1}{x_1+\delta}+\frac{1}{4\delta^2}\cdot\left[ln(x_2+\delta)-ln(\frac{k}{x_1+\delta})-(x_1+\delta)+(\frac{x_1-\delta}{x_1+\delta})\right])\cdot d(x_1x_2)$$
$$=(\frac{(ln\frac{y_1+2\delta}{y_1})}{2\delta}+\frac{\frac{(y_2+2\delta)ln(y_2+2\delta)-(y_2+\delta)}{(y_2)ln(y_2)-(y_2-\delta)}}{4\delta^2}+\frac{\frac{2(y_1)ln(|y_1+2\delta|)-(y_1+\delta)^2-2(ln(k)+\delta)(y_1+\delta)}{2(y_1)ln(|y_1|)-(y_1-\delta)^2-2(ln(k)+\delta)(y_1-\delta)}}{8\delta^2})$$
$$\cdot(\frac{(ln\frac{x_1+2\delta}{x_1})}{2\delta}+\frac{\frac{(x_2+2\delta)ln(x_2+2\delta)-(x_2+\delta)}{(x_2)ln(x_2)-(x_2-\delta)}}{4\delta^2}+\frac{\frac{2(x_1)ln(|x_1+2\delta|)-(x_1+\delta)^2-2(ln(k)+\delta)(x_1+\delta)}{2(x_1)ln(|x_1|)-(x_1-\delta)^2-2(ln(k)+\delta)(x_1-\delta)}}{8\delta^2})$$

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